The Effects of Stellar Collisions in Dense Environments

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ABSTRACT

Direct stellar collisions become an important component of the dynamics in extreme astrophysical environments. I explore the situations where such effects must be taken into account and discuss the physical processes relevant to collisional stellar systems. I introduce widely used modeling techniques, Fokker-Planck, smooth particle hydrodynamics, and full dynamical evolution models. The results of the modeling are presented, including an analysis of which physical processes dominante the dynamics and differences in the model results.

1. Introduction

The vast majority of stellar systems encountered in the Universe may be treated as collisionless, meaning direct physical collisions occur very infrequently and such collisions have a negligible affect on the overall dynamics of the system. In such situations, stars may be treated as pointlike particles that obey the collisionless Boltzmann equation (CBE). In a few extreme astrophysical environments, direct stellar collisions will influence the overall behavior, and it is these systems that I investigate in this paper. Environments in which large stellar densities are combined with high velocity dispersions can lead to important collisional effects, more quantitatively, the rate of direct stellar collisions is given by

$$\Gamma_{coll} = 16\sqrt{\pi} n \sigma r_\star^2 [1 + (\frac{v_\star}{2\sigma})^2] \quad (1)$$

where n is the number density of stars, σ is the velocity dispersion, $r_\star$ and $v_\star$ are the individual star's radius and escape velocity, respectively (2). One can invert this equation to determine the typical timescale for stellar collisions, $t_{coll} \approx \Gamma_{coll}^{-1}$. For a typical location in the disk of a spiral galaxy, this timescale is many orders of magnitude larger than a Hubble time. For the cores of dense globular clusters and galactic nuclei however, one might expect a non-negligible number of direct stellar collisions over the lifetime of the system. The collision timescales for a globular cluster core and a galactic nucleus are roughly, 1000 and 80 billion years, respectively. This is of order 100 times the age of a cluster and 8 times the age of a typical nucleus, but when one considers that these objects have upwards of 100,000 stars, even for a 1% probability, many stars will undergo direct collisions.

The dynamics of a system can become significantly altered when collisions become important, for example, direct collisions can lead to stellar mergers which can alter stellar evolution and lead to the formation of new stars when gas liberated during an encounter coalesces. Near collisions can cause an increase in gravitational relaxation and binary formation, and collisions of massive stars can lead to the formation of compact objects. Collisions involving two stars are the most common variety, but in the certain situations, binary-single and/or binary-binary interactions can have significant consequences. In section 2, I discuss the physical processes at work in stellar collisions. Section 3 deals with modeling techniques used to predict the behavior of such systems and section 4 covers the results of various models and their implication for the evolution of the system.

2. Physical Processes and Dynamical Effects

Equation (1) describes the rate at which direct collisions occur in a stellar system. This rate depends on the velocity dispersion and density, which leads to differing behavior in different regions. Systems with a low velocity dispersion tend to coalesce with negligible mass loss where as when the dispersion is comparable to or greater than the stellar escape speed (generally during later stages of cluster evolution), stars on grazing incidence trajectories tend to disrupt one another. Looking at the more common case when the stars merge directly, one would like to know something about the collision product. Insight can be gained by looking at the Kelvin-Helmholtz time scale for contraction

$$t_{KH} = \frac{GM^2}{Lr_\star} \cong 30(\frac{M}{M_\odot})^2[Myr] \tag{2}$$

In the low velocity dispersion situation, t_{KH} is typically less than t_{coll}, so a merger product will have time to contract back onto the main sequence (5). Additionally, the actual coalescence phase will tend to erase any memory of the star's pre-collisional evolution, such that it is ”reborn” after the merger.

Relaxation causes a cluster to develop a core/halo structure because of differences in the dispersion and the density at a given radial location. The relaxation time scale is

$$t_{relax} = \frac{v_{RMS}^3}{4(\frac{3}{2})^{0.5}\pi(Gm)^2 n \ln 0.4N} \tag{3}$$

where N is the total number of stars. Equation (3) illustrates the dependence on density and velocity dispersion, and these values are quite different in the core and halo due to the equipartition of energy which causes more massive stars to lose kinetic energy and fall towards the core, while less massive stars increase their velocity and move outward. It is this ”mass segregation” process the initiates the core/halo structure in a cluster, and leads to the core relaxing on smaller time scales than the halo. For example in a typical globular cluster, t_{relax} can vary by four to five orders of magnitude from the core to the outer regions (5).

Stellar evolution is another process that must be accounted for in models of collisional systems, as all but the low mass stars have experienced significant evolution over the timescales involved for collision and relaxation effects. For example, if one begins their simulation with normal, hydrogen burning main sequence stars, more massive stars will evolve off of the main sequence and produce stellar remnants (or explode leaving no remnant) on timescales much less than t_{coll}, such that the dynamics of the remnants becomes significant. Mergers in the low dispersion cases discussed above will assist the initial mass function by building more massive stars, in particular, in the core of a cluster where collisions are more frequent and high-mass stars (m $>$ 50 $M_\odot$ for present purposes) are readily created (5; 4). For the highest mass stars, evolutionary tracks become somewhat uncertain, for example: 1) in the 60 – 80 $M_\odot$ mass range, the phase of oxygen burning in the core is not entirely understood. Large cores will collapse to form black holes, where as intermediate cores will explode leaving no remnant, and smaller cores do not posses the potential energy to explode and go through a phase of violent pulsation prior to collapse to a less massive black hole. 2) there is an analog in the 80 – 300 $M_\odot$ range as stars with m $>$ 300 $M_\odot$ collapse to form a black hole, stars with 120 $M_\odot < m <$ 300 $M_\odot$ tend to explode without a remnant, and stars with 80 $M_\odot < m <$ 120 $M_\odot$ will pulsate and collapse to a lower mass hole. It is clear that the somewhat uncertain fate of high-mass stars formed in cores will introduce uncertainties in the model output. The final consideration of stellar evolution in these dense environments has to do with the role of gas liberated from a star either as a consequence of one of the disruptive encounters discussed above or a more natural endpoint of stellar evolution, such as a supernova explsion. Here, ejected gas in our two test environments has a very different behavior. In globular clusters, the ejecta escapes because it's velocity is typically much greater than the escape velocity of the cluster. Galactic nuclei are considerably more massive, and as a consequence, some appreciable fraction of the ejecta will cool and fall back, leading to further star formation (8; 6).

Binary heating can affect the dynamics of the system when near or direct collisions become important. This heating occurs as a star (single or binary) interacts with a binary system, and gains kinetic energy at the expense of the binary system's binding energy (the binary system becomes more tightly bound, but E_{TOT} is a constant, and the star goes away with a larger kinetic energy, increasing the overall ”temperature” of the stellar system). Another variant of binary heating can occur in dense environments when it is more probable to find three stars in the same small volume of space, at which point, two of the stars may become bound and their lowered energy is carried off by the third in the form of kinetic energy. Binary heating processes can continue until their semi-major axis is equal to the sum of their radii, at which point they would merge, or until the binary becomes so hard that any further interaction would result in it's ejection from the cluster (the ”hardness” of a binary system is a measure of the magnitude of its binding energy with respect to the mean kinetic energy in the cluster $\approx m\sigma^2$). These heating processes act as an energy reservoir to the system and during later stages of cluster evolution, may possibly be able to stabilize the core against collapse (2).

Finally, to get an accurate picture of the relevant physical processes, one must consider processes that are negligible during the majority of the cluster's life yet become significant in the later stages of core collapse when massive stars and remnants are common and densities are many orders of magnitudes greater than in the outer parts of the cluster. One needs to consider tidal disruption of stars by massive black holes that may have grown or the collisions between remaining main sequence stars and stellar remnants. Dissipation of energy though gravitational radiation during remnant-remnant encounters may become important, and could have interesting observational consequences (4).

3. Modeling Methods

One of the primary goals of computational astrophysics since the birth of modern computers has been to accurately model the dynamics of a large number of bodies. Initial attempts focused on predicting the trajectories of tens of stars (point-masses) moving in a spherically symmetric gravitational potential. In the last 30 years, the bulk of the work on cluster simulations has come from Fokker-Planck techniques (5; 2). These techniques include many of the physical processes listed above for 10^4 – 10^5 stars and treat individual collisions using smooth particle hydrodynamics (discussed below) (7; 8; 3). Recent increases in computing power have allowed more computationally expensive approaches such as modeling the entire system, not just the direct collisions using hydrodynamical techniques, and directly calculating the orbits of each star in a system (6).

3.1. Fokker-Planck

Fokker-Planck methods are begun by assuming a distribution function for the stars in the system, $f(\vec{r},\vec{v},t)$, where the position, velocity, and time variable are related by the total energy

$$E \;=\; \Phi(r,t) - \frac{1}{2}v^2 \tag{4}$$

and Φ is the gravitational potential (noting that E and Φ are greater than zero in this formulation). The distribution function, f, is then constrained to satisfy the collisional Boltzmann equation

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla} f - \vec{\nabla}\Phi\frac{\partial f}{\partial \vec{v}} \;=\; \Gamma[f] \tag{5}$$

where $\Gamma[f]$ is a term that accounts for collisions in the system. To perform numeric simulations of the cluster, one rearranges the collisional Boltzmann equation into the form

$$\frac{\partial f_i}{\partial t} + \frac{1}{p}\frac{\partial q}{\partial t}\frac{\partial f_i}{\partial E} \;=\; \frac{1}{p}\frac{\partial(m_i D_E f_i + D_{EE}\frac{\partial f_i}{\partial E})}{\partial E} - M - S \tag{6}$$

where the i subscript denotes the distribution function of ”test particles” with mass m_i. D_E and D_{EE} are the diffusion coefficients and dynamical friction coefficients, respectively. They contain all of the information about the direct stellar collisions that a test star may experience as well as the less direct scattering off of the overall stellar distribution. M is the combination of source and sink terms due to mergers, and S contains the source and sink terms due to stellar evolution. q and p are variables introduced to make the rewritten Boltzmann equation less visually taxing and correspond to

$$q(E,t) \;=\; \frac{1}{3}\int_0^{\Phi-E} r^2(2\Phi-2E)^{1.5}dr \quad and \quad p(E,t) \;=\; \frac{\partial q}{\partial E} \tag{7}$$

respectively. An additional term, LT could in principal be included to account for the higher order physical effects discussed above that become important at Later Times in the core’s evolution.

Equation (6) is then solved simultaneously with the Poisson equation

$$\nabla^2\Phi(r) \;=\; -4\pi G\sum_i \rho_i(r) \tag{8}$$

and these solutions are known as solutions to the Fokker-Planck equation. Fokker-Planck models need input values for f, Φ, and the mass density ρ. The most commonly used input conditions are taken from the Plummer Model.

$$f_i(E) \propto \frac{N_i R^2 R^{3.5}}{(GM)^5} \quad ;\rho_i(r) \propto \rho_o \frac{1}{(1+(\frac{r}{b})^2)^{2.5}} \quad ;\Phi(r) \propto \frac{GM}{\sqrt{r^2+b^2}} \tag{9}$$

3.2. Smooth Particle Hydrodynamics

Hydrodynamical effects must be taken into account during a stellar encounter (either direct collision or near miss). These effects become more pronounced when one or both of the participating stars is a binary. For example, binaries heating the core of a globular cluster have separations less than a few astronomical units (AU), and direct, stellar contact is likely in a binary-single star interaction and nearly unavoidable in binary-binary collisions (clearly, the point mass approximation is invalid for the case of two pairs of bound stars confined within a few AU when their radii are each a few percent of an AU) (3).

Smooth particle hydrodynamics assumes stars are composed of gaseous fluid elements, represented as ”quasi-particles” (Figure 1). To predict the global properties, the continuous values are interpolated between the particles, or ”smoothed”, and these local properties obey the equations of motion for a compressible fluid

$$\begin{aligned}
\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{v}) &= 0 \\
\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\vec{\nabla})\vec{v} &= -\frac{1}{\rho}\vec{\nabla}P-\vec{\nabla}\Phi \\
\rho\frac{\partial u}{\partial t}+\rho(\vec{v}\cdot\vec{\nabla})u+P\vec{\nabla}\cdot\vec{v} &= -\Lambda
\end{aligned} \tag{10}$$

where P is the pressure of the fluid and Λ is an energy loss term. Solving these equations based on an initial potential, Φ, and an equation of state, P, allow you to determine the properties of the fluid, which becomes information about the stellar system in this application. Such hydrodynamical models require input parameters including the total number of stars, the mass and orbital eccentricity of each, and impact parameters of collisional elements. Binary systems need additional inputs for their semi-major axes and orbital planes (8).

3.3. Recent Approaches

Fokker-Planck techniques have been the most widely used model of stellar systems, but they do have limitations. Fokker-Planck calculations are only soluble in an ”averaged” sense, meaning that the results are only meaningful when a large number of bodies is present (5; 6). More computing power in recent years has opened the door to new approaches, such as the full dynamical evolution of a discrete number of stars, keeping track of each star’s six-dimensional coordinates, $(\vec{r}_i, \vec{v}_i)$ throughout the simulation. In these models, when two stars pass ”too close” to one another (spatial separation $d \approx r_1$ or r_2), an independent algorithm declares the event a collision and uses a pre-calculated smooth particle hydrodynamical collision model appropriate to the incoming properties of the collision elements to determine the merger product or the change in their phase space coordinates if the stars do not merge. These approaches are applicable to a wide range of astrophysical environments due to the versatility of the program in dealing with different potentials and initial distributions. For example, direct dynamical evolution models are applicable to smoothly distributed potentials as well as potentials dominated by a single object (such as a supermassive black hole at the center of an active galaxy). These techniques even allow for the inclusion of General Relativistic effects that could become important in the inner portions of galactic nuclei (6). Inputs into these models includes the total number of stars, a gravitational potential, initial stellar mass spectrum, and an initial stellar distribution function, $f(E, L, L_z)$.

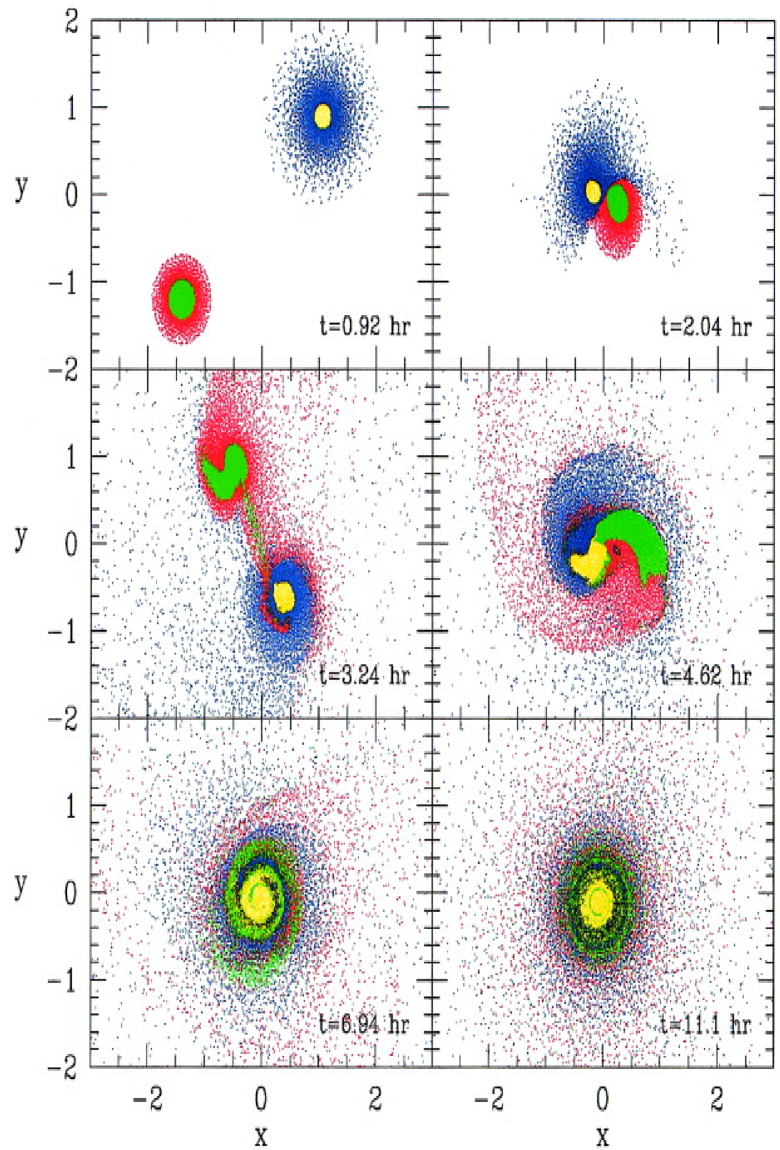

Fig. 1.— Smoothed particle hydrodynamical treatment of a direct stellar collision.

4. Results

In this section, I outline the results of the different modeling processes described above for various physical conditions and interaction scenarios. Results are presented for the interactions between individual binary stars and their effect on the evolution of globular clusters. Additionally, Fokker-Planck calculations concerning the evolution of star clusters in typical galactic nuclei and clusters of compact objects near the centers of galaxies are presented. Finally, I examine the results of more recent calculations of the inner regions of active galactic nuclei using a full dynamical evolution model.

Relaxation causes a loss of kinetic energy in the core of a globular cluster, as described in section 2. Without an additional source of energy, the core of a cluster should collapse, however binary heating may be able to convert enough of the system's gravitational potential energy into "heat" to avert, or at least postpone, core collapse (2; 8). Of particular interest recently is the role of binary-binary interactions, which, incidentally, have long been suspected as the primary source of runaway stars in the galactic disk (binary-binary interactions in open clusters can lead to a high-velocity ejection from the cluster) (1). Goodman and Hernquist present the possible outcomes of binary-binary interactions using the results of smooth particle hydrodynamical models (3). Possible scenarios include: a) all four stars coalesce, losing some gas from the new stellar envelope. b) one of the binaries becomes disrupted by a close encounter which excites tides between the members of system. These tides eventually cause one of the binaries to merge into a new star while the other binary system survives. c) one member of each binary system collide and merge, exerting strong gravitational tourques on the remaining two stars. Results from this point depend on initial conditions, but the most likely outcome is that one of the remaining two stars is disrupted enough so that it's orbit decays until it coalesces with first merger product, leaving a new binary system with the three-star merger product and one of the original binary members (Figure 2).

These results using the smooth particle hydrodynamics code were compared against similar models using a point-mass approximation, and it was found that until a physical collision took place, the point mass approximation was adequate in describing the behavior. They conclude that the dissapative effects are less dramatic than previously expected, and that binary-binary interactions seem to be incapable of providing enough kinetic energy to prevent core collapse, placing an upper limit on the core heating per collision of roughly 5 $m_\star(10\ \mathrm{km/s})^2$. The authors note, however, that in a high density core, the high frequency of mergers coupled with the tendency for the most massive stars to reside there should make it possible for massive stars to undergo multiple mergers on less than nuclear timescales, thereby accelerating stellar evolution in the core and perhaps slowing the collapse.

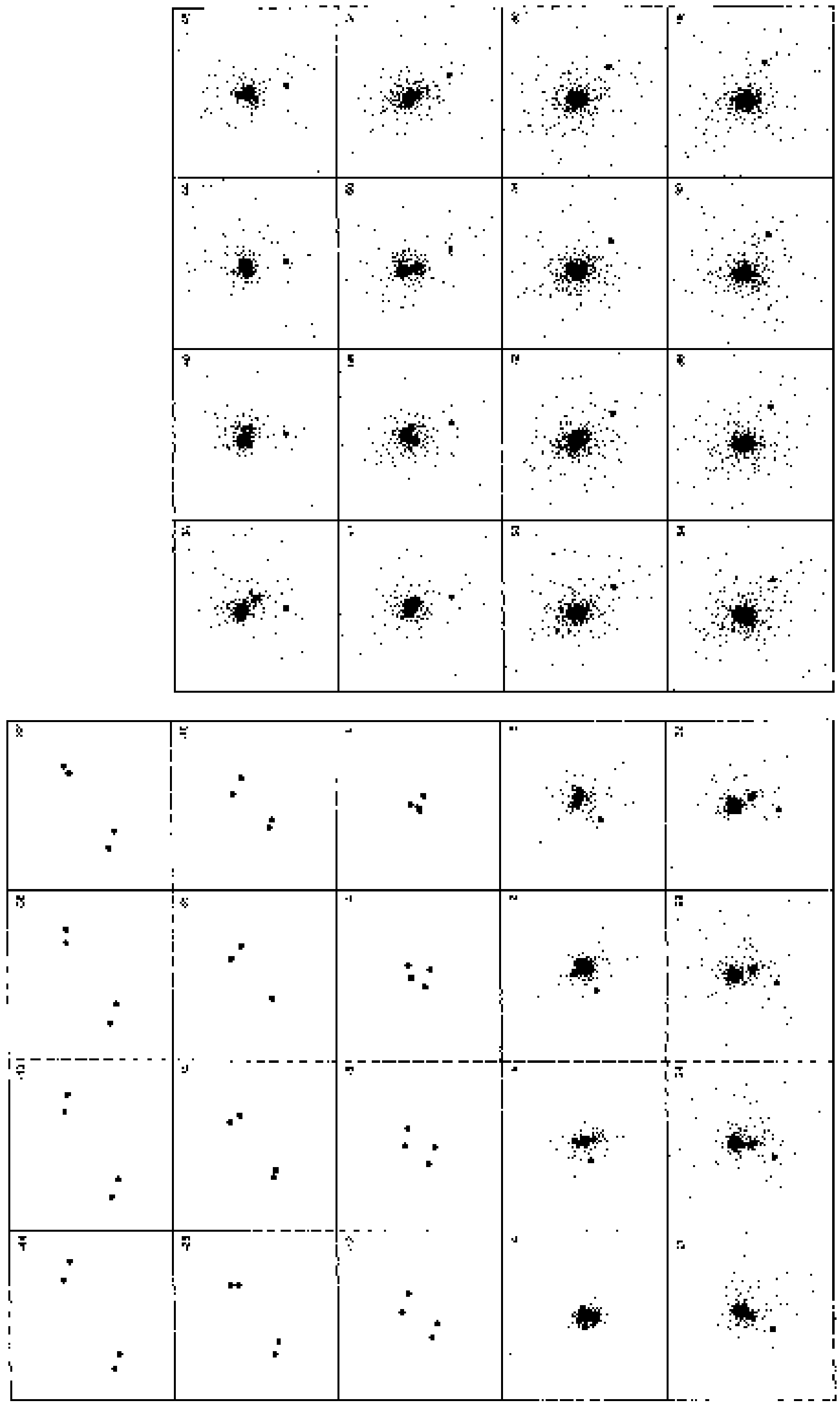

Fig. 2.— Smooth particle hydrodynamical simulations of a binary-binary interaction (scenario ”c)” described above).

4.1. Cluster of Stars: Fokker-Planck analysis

Quinlan and Shapiro have studied the behavior of dense star clusters in galactic nuclei using Fokker-Planck methods to determine the effects that dominante the dynamics in such situations (5). They find the mass segregation induced by the stars attempts to reach equipartition (as discussed above) to be a large effect, and in cases where the more massive stars cannot reach thermal equilibrium with their less massive neighbors, they tend to form a rapidly evolving, self-gravitating core. The lower mass halo stars act as a reservoir to remove more and more energy from the high-mass members of the core. The overall effect they find from models with various initial conditions is that simulations beginning with multi-mass stellar systems tend to evolve more rapidly than models with a single initial mass that form massive stars through mergers. They also find that relaxation can incite multiple mergers, leading to the formation of stars with mass greater than 100 $M_\odot$ to be formed, and mass segregation causes these stars to sink to the center of the core before a significant increase in the velocity dispersion can occur, favoring further collisions and producing intermediate-mass (10^2 – 10^3 $M_\odot$) black holes at the cluster's center.

Simulations were run with a number of physical processes ”turned off” to determine the effects of each, particularly, the role of star formation and the dependence on initial density. In models without star formation, mass loss due to stellar evolution was assumed to be ejected from the cluster. These models find a quick buildup of massive stars ($m > 100$ $M_\odot$) at the core due to collisions, with many neutron stars forming from the evolution of 8 – 16 $M_\odot$ stars (Figure 3). These models experience some problems and uncertainties, as mass segregation leads to a small number of the most massive stars in the core, and as discussed before, Fokker-Planck methods are generally valid only for ”large N” systems. Other problems encountered by these models are that as high mass stars are formed through mergers, eventually the lifetime of the stars becomes significantly shorter than the collision time, which clearly leads to an unphysical paradox of how to continue to grow more massive stars if the building blocks have disappeared. The authors ascribe this problem as being due to an algorithm in the code that resets the age of a merger product to zero following a collision. The main result from models without star formation included, however, is that there is a strong dependence on the initial density of the model used. In models with an initial density of three times less than used in the results described above, no stars with a mass greater than 30 $M_\odot$ were formed and core collapse was reversed.

Further models that included the effects of star formation were performed, and the deviations from the models without star formation examined. New stars are formed out of the stellar evolution products (ejected gas), and in the case of high initial density, massive stars are still formed and mass segregation still dominantes (Figure 4). In the case of lower initial densities though, the losses from stellar evolution that are retained in these models allows core collapse to continue beyond the point where it is halted in models without star formation. Given the correct initial conditions, it seems that high mass objects (both stars and remnants) and core collapse is the preferred endpoint of dense star clusters in galactic nuclei.

4.2. Cluster of Compact Objects: Fokker-Planck analysis

One sees that it is feasible to form massive compact objects in the core of clusters, and the next step is to determine the dynamics of such a cluster of compact objects. The jump to a cluster of massive, compact objects is somewhat forced as we saw a small number of objects remaining in the core near collapse, but by

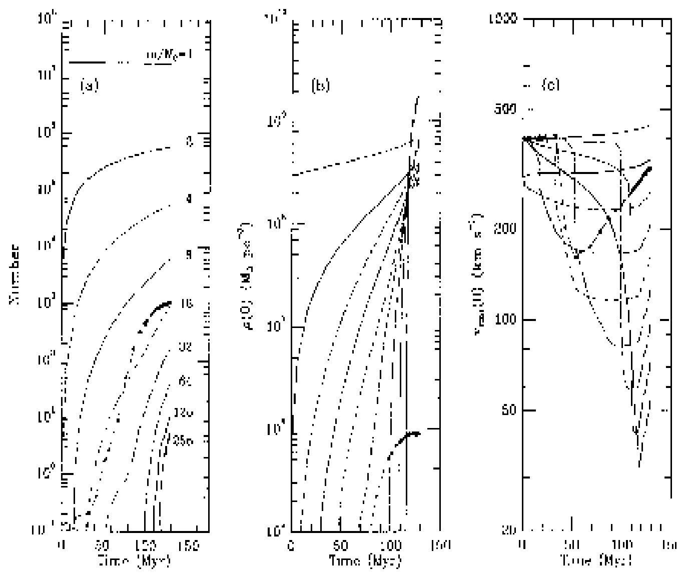

Fig. 3.— Fokker-Planck simulations of a stellar cluster in a galactic nucleus without continued star formation.

assuming that a substantial population of such objects exists in the core, one may apply a similar Fokker-Planck analysis modified by replacing the stellar collisions with collisions between compact objects. Such an analysis was carried out again by Quinlan and Shapiro as a follow-up to the paper in which they determine that massive objects are readily grown in dense cluster cores (4). The dominant physical process in a cluster of neutron stars and black holes are interactions with binary stars as this effectively increases the collisional cross-section to something like πa^2, where a is the semi-major axis of the binary, potentially orders of magnitude larger than the single star cross-section of πr_{NS}^2, which is very small for typical neutron star radii. Binary systems where both members are compact objects are also interesting because they may decay through the emission of gravitational radiation and coalesce. An environment rich in compact object binaries should be the most promising place to detect gravitational radiation with future laser-interferometric observatories.

Mass segregation is again a dominant process as the neutron stars and black holes merge to form more massive objects. Even with the effects of mass segregation ”turned off”, massive objects form at the center preferentially because of the relaxation arguments presented in section 2, and core collapse progresses quickly (Figure 5). If mergers are turned off in a given model, but mass segregation is allowed to progress, the highest initial mass objects fall to the center, and even though they cannot merge, a build-up of the most massive objects occurs and the core collapses quickly. Although the Fokker-Planck methods cannot follow core collapse indefinitely, the authors postulate that the trend towards building fewer and fewer objects with higher and higher masses will continue until the core of the cluster is dominated by a central, supermassive compact object (black hole).

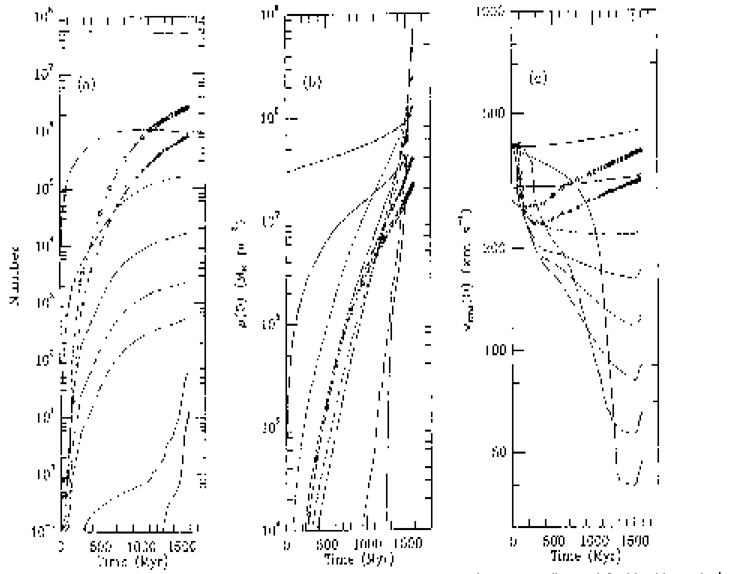

Fig. 4.— Fokker-Planck simulations of a stellar cluster in a galactic nucleus with star formation and a Salpeter birthrate.

4.3. Other Environments: New Approaches

Using the recent advances in computing power, it is possible to model more exotic astrophysical environments and compare results of the more sophisticated techniques with traditional methods, such as Fokker-Planck analysis. Rauch has modeled a stellar population orbiting a central, massive black hole using techniques that follows the phase-space position of each star, including collisions, disruptions, and deflections throughout the simulation (6). He follows the orbits using the geodesic equations of the Kerr metric, and can compare the results to the non-relativistic Keplerian predictions to determine if there may be any effects of general relativity that could be observed near the center of active galaxies. This situation differs qualitatively from the environments considered above in that as collisions will play a large role in the dynamics, but at the high velocity dispersions around a supermassive black hole, collisions will be more likely to destroy the stars than lead to mergers.

Models for a variety of initial conditions were performed and stellar gas liberated during the encounters was the largest collision product, and results show that roughly half of this gas will be ejected from the system and half will eventually be accreted onto the black hole. Similarly, of stars whose orbits led them close enough to the central object to become tidally disrupted, about half of the liberated material fell onto the black hole. Another process that led to the elimination of stars in these simulations were encounters with other stars that led to large changes in the angular momentum of a given orbit. These stars tended to be ejected from the system or directly plunge into the black hole. It should be noted however, that while more mass per star may be lost from stars plunging into the hole, much more radiative energy is released when the star is disrupted and the black hole accretes the liberated gas.

Results of these simulations are practically indistinguishable from a non-relativistic treatment of a stellar population orbiting a supermassive central object. This is not entirely unexpected as the relativistic effects become greatest closest to the hole (radius of marginal stability $\approx 6\ R_g$ for a Kerr black hole), and

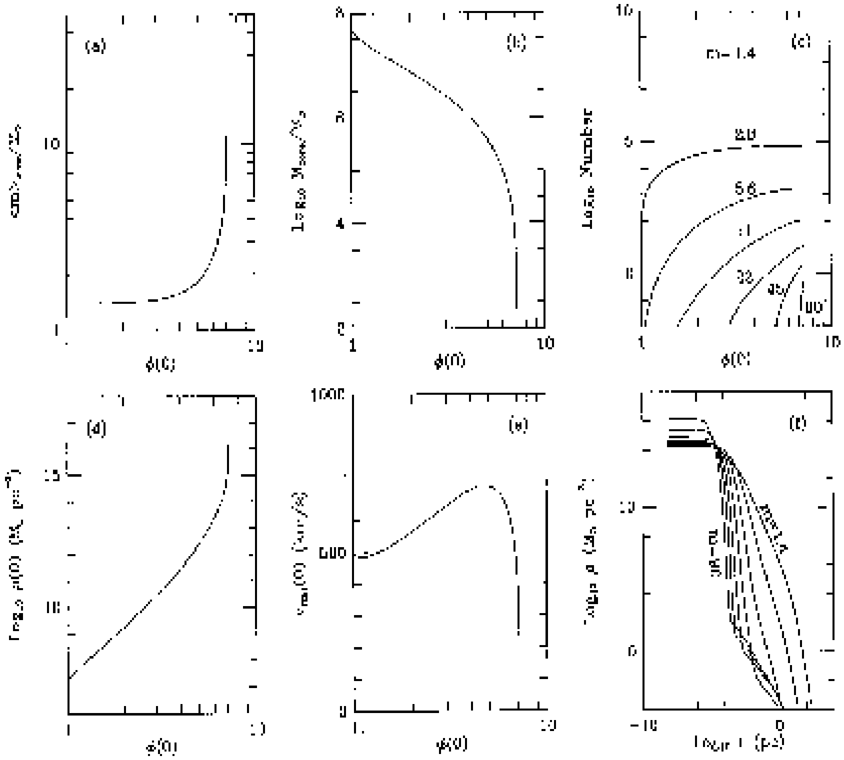

Fig. 5.— Fokker-Planck simulations of a cluster of compact objects including effects of mergers, mass segregation, and gravitational radiation.

the stars in the model became tidally disrupted before they could approach too close (typical tidal radii in these models fell between a few tens and one hundred R_g). These results are very similar to the Fokker-Planck results with one major difference: in these simulations, the core tends to a constant density in the steady state limit where as Fokker-Planck simulations predict an $r^{-1/2}$ relationship in the inner, collision dominated region. The author suggests that this is most likely due to the approximations made in the Fokker-Planck approach, particularly the isotropy assumed in the density distribution (ie- all effects at a radius *r* are averaged together).

5. Closing

Direct stellar collisions are negligible in the majority of astrophysical environments. Dense concentrations of stars such as the cores of globular clusters and clusters in galactic nuclei can produce situations where direct stellar collisions and their merger products can have a substantial effect on the dynamics of the system. In this paper, I reviewed the relevant physics affecting collisional stellar systems, such as mergers, relaxation, stellar evolution, the formation of new stars, and binary heating. Numerical modeling techniques were reviewed, traditional Fokker-Planck analysis and more recent methods such as smooth particle hydrodynamics and direct dynamical evolution models. Finally, results from the different models were presented and reflected a strong dependence on initial conditions. The general trend was that collisions helped ”grow” massive stars and compact objects and that generally, core collapse could not be averted. As computing capability increases in the future, our ability to directly follow the phase-space orbits of stars will allow for true N-body simulations with proper treatment of general relativistic effects. These results can be compared with the first observations of gravitational radiation obtained by the new generation of interferometric observatories coming online in the next decade.

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